100% MIDDLE SCHOOL MATHEMATICS CHALLENGE



10 PRACTICE TESTS WITH FULL DETAILED SOLUTIONS

Developing Math Concepts Through Math Contest Preparation

American Mathematics Contest

AMC8 - MathCON - Math Leaugues





Practice Exam 2

- ♦ You have **75 minutes** for **25 problems**.
- There are no penalties for incorrect answers. Answer as many problems as you can; return to the others in the time you have left for the test.

Problem 1. If $x + 4 = y^2 - 1 = z^2 + 2 = t - 3 = m^2 + 12$, which of the numbers x, y, z, t, and m is the greatest?

A) x B) y C) z D) t E) m

Problem 2. Suppose a and b are integers and a + b is an odd number. Which of the following is always true?

I) a - 2b is even II) $a \cdot b$ is even III) 4a + b is even

	A) Only I	B) Only II	C) Only III	D) I and II	E) I, II and III
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Practice Exam 2

Problem 3. How many ways can the letters of the word TRIANGLE be arranged such that the letters ANGLE appear consecutively, and in that order?

	A) 6		B) 18		C) 20		D) 24		E) 56	
Prob	lem 4.	What is t	he area of	an iso	osceles	triangle	with side	lengths	10, 10, a	and 12?

A) 48 B) 50 C) 60 D) 72 E) 96

Problem 5. In an online math practice test, Junaid attempts exactly $\frac{3}{4}$ of the problems and answers $\frac{5}{8}$ of those problems correctly. When he submits the test, he finds that he answered 105 problems correctly. How many math problems were on this test?

A) 220 B) 224 C) 243 D) 248 E) None of the preceding

Assessment for Practice Exam 2

Q #	Торіс	YA	СА				Notes
1	Algebra						
2	Number Theory						
3	Combinatorics						
4	Geometry						
5	Algebra						
6	Number Theory						
7	Combinatorics						
8	Geometry						
9	Algebra						
10	Number Theory						
11	Combinatorics						
12	Geometry						
13	Algebra						
14	Number Theory						
15	Combinatorics						
16	Geometry						
17	Algebra						
18	Number Theory						
19	Combinatorics						
20	Geometry						
21	Algebra						
22	Number Theory						
23	Combinatorics						
24	Geometry						
25	Algebra						
CA: Co	orrect Answer YA	: Your A	nswer	☑ Co	rrect	⊠I	ncorrect

Solutions for Practice Exam 2

Problem 1. If $x + 4 = y^2 - 1 = z^2 + 2 = t - 3 = m^2 + 12$, which of the numbers x, y, z, t, and m is the greatest?

A) x B) y C) z D) t E) m

Solution. As $y^2 - 1 = z^2 + 2 = m^2 + 12$, the greatest value among y^2 , z^2 and m^2 is y^2 . On the other hand, as x + 4 = t - 3, the greatest value among x and t is t. Thus, when we compare t and y^2 by using $t - 3 = y^2 - 1$, we have $t = y^2 + 2$. That is, t is the greatest among all of them. The answer is D.

Problem 2. Suppose a and b are integers and a + b is an odd number. Which of the following is always true?

I) a - 2b is even II) $a \cdot b$ is even III) 4a + b is even

A) Only I B) Only II C) Only III D) I and II E) I, II and III

Solution. Since a + b is an odd number then a and b both cannot be odd or even at the same time. This lead us to decide $a \cdot b$ is always an even number since odd time even gives us always an even number. The other two options, I and III, don't always create an even number. The answer is B.

Problem 3. How many ways can the letters of the word TRIANGLE be arranged such that the letters ANGLE appear consecutively, and in that order?

A) 6 B) 18 C) 20 D) 24 E) 56

Solution. We consider ANGLE as a one piece. Then there are 4 pieces to arrange. Therefore there are 4! = 24 ways. The answer is \boxed{D} .

Problem 4. What is the area of an isosceles triangle with side lengths 10, 10, and 12?

A) 48 B) 50 C) 60 D) 72 E) 96

Solution. Using Heron's area formula for triangles, the area of the triangle with sides 10, 10 and 12 is

 $\sqrt{16 \cdot (16 - 10) \cdot (16 - 10) \cdot (16 - 12)} = \sqrt{16 \cdot 6 \cdot 6 \cdot 4} = 48.$

The answer is A.

Problem 5. In an online math practice test, Junaid attempts exactly $\frac{3}{4}$ of the problems and answers $\frac{5}{8}$ of those problems correctly. When he submits the test, he finds that he answered 105 problems correctly. How many math problems were on this test?

A) 220 B) 224 C) 243 D) 248 E) None of the preceding

Solution. Letting N be the number of problems in the test, Junaid attemps $\frac{3N}{4}$ problems where $\frac{5}{8} \cdot \frac{3N}{4} = \frac{15N}{32}$ of them are correct. Since he answered 105 problems correctly, i.e. $105 = \frac{15N}{32}$, we have N = 224. The answer is B.

Problem 6. Distinct, nonzero digits A, B, and C are such that the three-digit numbers ABC, CAB, and BCA are divisible by 4, 5, and 9, respectively. What is the greatest possible value of $A \times B \times C$?

A) 20 B) 180 C) 200 D) 210 E) 240

Solution. Since CAB is divisible by 5, the value of B is either 0 or 5. However, BCA is a three digit number, therefore B = 5. On the other hand, since ABC is divisible by 4, the value of BC is either 52 or 56. Finally, since BCA is divisible by 9, the value of A is either 2 or 7. Therefore, the greatest value of $A \cdot B \cdot C$ is $7 \cdot 5 \cdot 6 = 210$. The answer is D.

Problem 7. Set $A = \{-7, -6, -5, -4, -3, -2, -1, 1, 2, 3\}$. What is the probability that product of two randomly selected numbers is positive number?

A) $\frac{1}{15}$ B) $\frac{4}{15}$ C) $\frac{7}{15}$ D) $\frac{8}{15}$ E) $\frac{14}{15}$

Solution. One can choose two random numbers from A in C(10, 2) = 45 ways. Their product is positive when both of the numbers are positive or negative at the same time. That is, there are C(7, 2) + C(3, 2) = 21 + 3 = 24 such choices. Thus, the probability is $\frac{24}{45} = \frac{8}{15}$. The answer is D.

Problem 8. A rectangular box has integer side lengths in the ratio of $1 : \frac{3}{2} : 2$. Which of the following could be the volume of the box?

A) 136 B) 148 C) 160 D) 192 E) 204

Solution. The lowest integer ratio corresponding to $1:\frac{3}{2}:2$ is 2:3:4, i.e. the sides of the box are 2a, 3a and 4a for some integer a. Then the volume of the box is $24a^3$. Hence, it would be 192 for a = 2, while the others are not in the form $24a^3$. The answer is D.

Problem 9. If $A = \frac{21}{19} + \frac{11}{29}$, then which of the following equals $\frac{18}{29} - \frac{2}{19}$? A) 2 - A B) 1 - A C) A D) A + 1 E) A + 2

Solution.

$$\frac{18}{29} - \frac{2}{19} = \left(1 - \frac{11}{29}\right) - \left(\frac{21}{19} - 1\right) = 2 - \left(\frac{11}{29} + \frac{21}{19}\right) = 2 - A.$$

The answer is A.

Problem 10. The first page number of a book is 1. The sum of page numbers in the book is less than 2020. If there were 1 more page, then the sum of page numbers in the book would be more than 2020. Find the number of pages of the book.

A) 59 B) 60 C) 61 D) 62 E) 63

Solution. Assume n is the last page of the book. Then we have

$$1 + \dots + n = \frac{n \cdot (n+1)}{2} < 2020 \text{ and } 1 + \dots + (n+1) = \frac{(n+1) \cdot (n+2)}{2} > 2020.$$

Since n is an integer, we have $n \le 63$ in the first inequality, and $n + 1 \ge 64$ in the second inequality. Thus n = 63. The answer is E.

Problem 11. Alice and Bob each roll a fair 12-sided die. What is the probability that Alice's roll is greater than or equal to Bob's roll?

A)
$$\frac{11}{24}$$
 B) $\frac{1}{2}$ C) $\frac{25}{48}$ D) $\frac{13}{24}$ E) $\frac{7}{12}$

Solution. Let (a, b) be the tuple where the value of Alice 's die is a and the value of Bob's die is b. There are $12 \cdot 12 = 144$ such tuples since $a, b \in \{1, 2, ..., 12\}$. The number of tuples where $a \ge b$ are

$$\#(a \ge 1) + \#(a \ge 2) + \dots + \#(a \ge 12) = 12 + 11 + \dots + 1 = 78.$$

Therefore, the probability that Alice's roll is greater than or equal to Bob's roll is $\frac{78}{144} = \frac{13}{24}$. The answer is D.