BOOK'S TARGET CONCEPTS AND SKILLS

This Math Fluency book contains Pre & Post tests, two benchmark tests, 22 chapters with topic explanations, 200 examples and problems, and 40 problem sets with 1000 problems. Overall it covers the following list of concepts and skills essential for success in middle school mathematics and algebra courses.

Chapter 1

- Skill 1 Positive and negative integers
- Skill 2 Odd and even numbers
- Skill 3 Order of operations
- Skill 4 Associative property
- Skill 5 Distributive property

Chapter 2

- Skill 6 Negative and positive integers
- Skill 7 Integer operations
- Skill 8 Odd and even numbers
- Skill 9 Evaluating expressions
- Skill 10 Additive and multiplicative inverses
- Skill 11 Consecutive numbers and sums

Chapter 3

- Skill 12 Prime numbers & composite numbers
- Skill 13 Prime factorizations
- Skill 14 Divisors & Factors
- Skill 15 Perfect squares & perfect cubes
- Skill 16 Divisibility Rules

Chapter 4

- Skill 17 Divisors / factors of numbers
- Skill 18 Multiples of numbers
- Skill 19 Least Common Multiple (LCM)
- Skill 20 Greatest Common Divisors (GCD)

Chapter 5

- Skill 21 Equal fractions and fraction types
- Skill 22 Comparing fractions
- Skill 23 Adding and subtracting fractions
- Skill 24 Multiplying fractions
- Skill 25 Dividing fractions

Chapter 6

- Skill 26 Defining the percent and percentage of a number.
- Skill 27 Defining decimal numbers.
- Skill 28 Conversion between decimals, fractions, and percents.

Chapter 7

- Skill 29 Variables
- Skill 30 Evaluating an expression
- Skill 31 Adding and subtracting expressions
- Skill 32 Combining complex expressions
- Skill 33 Simplifying rational expressions

Chapter 8

- Skill 34 Setting up an equation
- Skill 35 Solving simple equations
- Skill 36 Solving multi steps equations
- Skill 37 Solving simple rational equations

Chapter 9

- Skill 38 Angles
- Skill 39 Areas of basic polygons
- Skill 40 Perimeters of basic polygons
- Skill 41 Volumes of basic 3-D shapes
- Skill 42 Setting equations by using geometric figures

Chapter 11

- Skill 43 Solving equations involving decimals
- Skill 44 Solving equations involving rational expressions
- Skill 45 Holistic equation solving- Cover-up method
- Skill 46 Absolute values
- Skill 47 Solving absolute value equations
- Skill 48 Inequalities
- Skill 49 Solving linear inequalities

Chapter 12

- Skill 50 Defining ratios and proportions
- Skill 51 Solving proportion equations
- Skill 52 Solving real-life problems with ratios and proportions
- Skill 53 Applying ratios & proportions to similar figures

Chapter 13

- Skill 54 Defining functions & binary operations.
- Skill 55 Representing linear functions in various forms.
- Skill 56 Solving equations involving functions & binary operations.
- Skill 57 Applying linear equations to real-life situations.

Chapter 14

- Skill 58 Defining exponential numbers.
- Skill 59 Defining the square root of a number.
- Skill 60 Using the rules of exponents and square roots for simplifying expressions.
- Skill 61 Using the rules of exponents and square roots for solving equations.

Chapter 15

- Skill 62 Basic properties of rectangle and square
- Skill 63 Angle properties of a triangle
- Skill 64 Side properties of a triangle
- Skill 65 Special right triangles
- Skill 66 The Pythagorean Theorem
- Skill 67 Triangle similarities

Chapter 16

- Skill 68 Defining the coordinate plane.
- Skill 69 Applying the Distance & Midpoint Formulas in the coordinate plane.
- Skill 70 Applying the rules of reflection in the coordinate plane.
- Skill 71 Area &Perimeter in the coordinate plane

Chapter 17

- Skill 72 Defining prisms.
- Skill 73 Finding the surface area and volume of prisms.
- Skill 74 Finding the circumference, area, arc length, and sector area of a circle.
- Skill 75 Finding the surface area and volume of a cylinder.

Chapter 18

- Skill 76 Defining linear equations in one/two variables.
- Skill 77 Solving linear equations.
- Skill 78 Defining linear functions.
- Skill 79 Graphing linear functions.

Chapter 19

- Skill 80 Defining the decimal system.
- Skill 81 Defining number sets and their connections.
- Skill 82 Prime numbers and prime factorization.
- Skill 83 Converting a repeating decimal to a fraction.
- Skill 84 Rationalizing a denominator

Chapter 20

- Skill 85 Defining main measures of central tendency.
- Skill 86 Finding main measures of central tendency for a given data set.
- Skill 87 Using various frequency charts to represent different data types.

Chapter 21

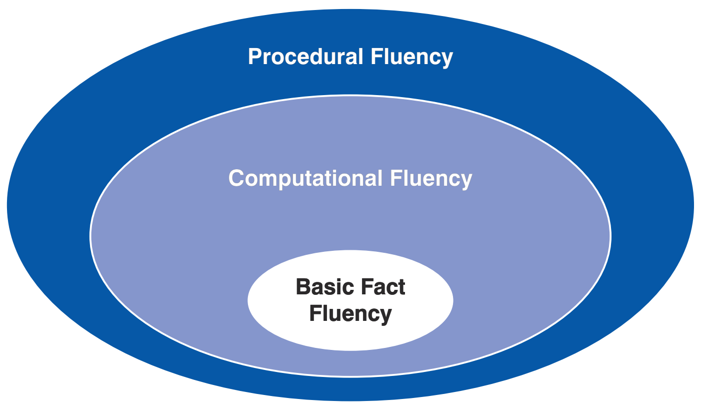
- Skill 88 Defining sets and subsets.
- Skill 89 Defining union and intersection of sets.
- Skill 90 Defining the difference set.
- Skill 91 Applying the Inclusion-Exclusion Principle to real-life situations.

What is Procedural Fluency?

NCTM Position

Procedural fluency is a critical component of mathematical proficiency. Procedural fluency is the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another. To develop procedural fluency, students need experience in integrating concepts and procedures and building on familiar procedures as they create their own informal strategies and procedures. Students need opportunities to justify both informal strategies and commonly used procedures mathematically, to support and justify their choices of appropriate procedures, and to strengthen their understanding and skill through distributed practice (National Council of Teachers of Mathematics (NCTM), 2014).

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving.



Procedural fluency encompasses both basic fact fluency and computational fluency.
This figure is adapted from (Bay-William et.al 2022).

Three components of procedural fluency defined by NCTM (2014) are as follows:

1. Efficiency
2. Flexibility
3. Accuracy

Principles and Standards for School Mathematics states, “Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate *flexibility* in the computational methods they choose, *understand* and can explain these methods, and produce accurate answers efficiently. The computational methods that a student uses should be based on mathematical ideas that the student understands well, including the structure of the baseten number system, properties of multiplication and division, and number relationships” (p. 152).

PRE-TEST

Math Fluency Pre-Test & Evaluation

1. Compute: 86 – 72 ÷ 8 × 3

2. What is the sum of the quotient and remainder when you divide 7550 by 15?

3. What number should go in the blank so that the equation below will be true?

______ × 90 = 30

4. What number should go in the blank so that the equation below will be true?

8 ÷ ______ = 40

5. What is 15% of 40?

6. Find an integer between 100 and 150 that is divisible by 7.

7. Find the greatest common factor of 28, 42, and 56.

8. Find the largest prime factor of 72.

9. Solve for x: (x – 9) ÷ 4 = 12

10. Evaluate 14 × 8.9 + 86 × 8.9

Math Fluency Pre-Test & Evaluation

Problem	Topic	My Answer	Correct Answer	Notes
1	Order of operations			
2	Long division			
3	Multiplication			
4	Division			
5	Percent			
6	Divisibility			
7	GCD/LCM			
8	Prime factors			
9	Equations			
10	Distributive property			
11	Integers			
12	Fractions			
13	Arithmetic manipulations			
14	Equations			
15	Geometric equations			
16	Fraction multiplications			
17	Fraction Divisions			
18	Number Lines			
19	Divisibility			
20	Functions			
21	Ratios and Proportions			
22	Inequalities			
23	Sets			
24	Coordinate Geometry			
25	Statistics and Data			
26	Radicals			
27	Prime factorizations			
28	Fraction Divisions			
29	Complex Equations			
30	Geometry			

CHAPTER 1

Whole Numbers & Operations

Target Concepts and Skills

- ☐ Positive and negative integers
- ☐ Odd and even numbers
- ☐ Order of operations
- ☐ Associative property
- ☐ Distributive property

Definition 1.1 – Natural Numbers.

$\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$ The natural numbers are also called the counting numbers.

Definition 1.2 – Whole Numbers.

Whole numbers are positive numbers, including zero, without any decimal or fractional parts. They are numbers that represent whole things without pieces. The set of whole numbers is represented mathematically by the set

$\mathbb{W} = \{0, 1, 2, 3, 4, 5 \ldots\}$

Definition 1.3 – Integers.

Any number that is not a fraction or decimal.

- Positive integers, $\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$,
- Negative integers, $\mathbb{Z}^- = \{\ldots, -3, -2, -1\}$,
- Integers, $\mathbb{Z} = \mathbb{Z}^- \cup \{0\} \cup \mathbb{Z}^+$, 0 has no sign.

Definition 1.4 – Odd and Even Numbers

An odd number is an integer which is not a multiple of two. An integer that is not an odd number is an even number.

Example 1.1

- $2c + 3$ is always odd.
- If $2a + b$ is odd, then b is odd.

Definition 1.5

A digit is a written symbol for any of the ten numbers from 0 to 9.

Problem 1.1

If a and b are distinct digits, then what is the maximum value of $5a + 6b$?

Solution 1.1

In order to create the largest value we replace 9 with b and 8 with a. So,

$5(8) + 6(9) = 40 + 54 = 94.$

Remark 1.1 – Order of Operations

First of all, parentheses must be performed (but some exceptions) and exponents are next. Division or multiplication have the same priority, and addition or subtraction have the same priority. Again, we need to use the following conventions on order of operations.

- All powers (exponents) are considered first
- Multiplication and division are considered from left to right.
- Addition and subtraction are considered from left to right.

Problem 1.2

Evaluate $5 - 8 \div 4 \times (3 - 1)$

Solution 1.2

By the order of operations

$5 - \underbrace{8 \div 4}_{\text{even}} \times (2)$

$= 5 - 2 \times 2$ (from left to right)

$= 5 - 4 = 1$

Problem 1.3

Evaluate

$35 - 10 \div 2 \times 5 + 3$

Solution 1.3

$35 - 10 \div 2 \times 5 + 3$

$= 35 - 5 \times 5 + 3$

{division and multiplication working from left}

$= 35 - 25 + 3$

{subtraction and addition working from left}

$= 10 + 3 = 13$

Remark 1.2 – PEMDAS - Distributive & Associative Prop

For example, $24\times12-23\times12$, we see that students just memorize the PEMDAS mnemonic they learned. They could start with multiplication and then subtraction which takes additional step. Instead, knowing appropriate number properties such as the associative property of multiplication or, the distributive property of multiplication over addition subtraction would be more efficent.

Problem 1.4

Simplify the following:

a) $3\times\left(128\times\frac{1}{3}\right)$

b) $81\times52-71\times52$

c) $25\times(13\times4)$

Solution 1.4

a) Using associative and commutative properties of multiplication,

$$3\times\left(128\times\frac{1}{3}\right) = 3\times\left(\frac{1}{3}\times128\right) = \left(3\times\frac{1}{3}\right)\times128 = 1\times128 = 128$$

b) Using the distributive property of multiplication over subtraction

$$81\times52-71\times52 = (81-71)\times52 = 10\times52 = 520$$

c) Using associative and commutative properties of multiplication

$$25\times(13\times4) = 25\times(4\times13) = (25\times4)\times13 = 100\times13 = 1300$$

Problem 1.5

Compute $61 \cdot 9 + 61 \cdot 91$

Solution 1.5

Instead of separately computing $61 \cdot 9$ and $61 \cdot 91$, we can use the distributive property:

$61 \cdot 9 + 61 \cdot 91 = 61 \cdot (9 + 91)$

$= 61 \cdot 100 = 6100.$

Problem 1.6

What is $24 \cdot 16 \cdot 28 \div (12 \cdot 8 \cdot 14)$?

Solution 1.6

Instead of multiplying first top and bottom number, we can bring similar number together

$$\left(24\cdot\frac{1}{12}\right)\left(16\cdot\frac{1}{8}\right)\left(28\cdot\frac{1}{14}\right) = \left(\frac{24}{12}\right)\cdot\left(\frac{16}{8}\right)\cdot\left(\frac{28}{14}\right)$$

Problem 1.7

Compute $(128 \cdot 4) \div (128 \cdot 32)$

Solution 1.7

We convert the division to multiplication

$$128 \cdot 64 \cdot \frac{1}{128\cdot32} = \left(128\cdot\frac{1}{128}\right)\cdot\left(64\cdot\frac{1}{32}\right)$$

$$= \left(\frac{128}{128}\right)\cdot\left(\frac{64}{32}\right) = 1 \cdot 2 = 2$$

1. $2 \times 4^2 - (8 \div 2)$

2. What is the value of
$2 \times 0 \times 1 + 1$?

3. Compute:
$2 + 2 \times 2 - 2 \div 2 + 2$

4. Compute:
$-2 \times (6 \div 3)^2$

5. Which of the following is a multiple of 5?
a) $1 \times 2 + 3 + 4$　b) $1 + 2 \times 3 + 4$
c) $1 + 2 \times 3 \times 4$　d) $1 \times 2 \times 3 \times 4$

6. If $\frac{8 + a}{20} = \frac{1}{2}$
What is a?

7. Which of the following is correct?
a) $1 \times 8 + 8 \times 1 = 18$
b) $0 \times 9 + 9 \times 0 = 18$
c) $2 \times 7 + 7 \times 2 = 28$
d) $3 \times 6 + 6 \times 3 = 18$

8. Evaluate
$25 \times (13 \times 4)$.

9. Evaluate
$5 - 3 \times 4^3 \div (7 - 1)$

10. Compute:
$76 - 72 \div 8 \times 3$

11. What is the sum of all two-digit multiples of 9?

12. Compute

$4(101 + 103 + 105 + 107 + 109 + 111 + 113 + 117 + 119)$

13. Which of the following is the sum of three consecutive integers?

A) 14 B) 7 C) 26 D) 27 E) 38

14. Let n be a positive integer.

If $(1 + 2 + 3 + 4 + 5)^2 = 1^3 + 2^3 + \ldots + n^3$

What is n?

15. What is the digit in the ones place of the result of multiplying?

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9$

16. What is the largest value of $a \times b^c$.

When we replace 2, 3, and 4 with a, b, and c once.

17. Compute:

$(3^3 - 3) \div 4 \times 6 \div 1$

18. To make the statement $(10 \,?\, 5) + 4 - (10 - 9) = 5$ true, what operation should be replaced for?

19. If a, b are distinct digits, then what is the maximum value of 7a + 5b?

20. $A = 26 \times 301$.

$B = 13 \times 601$.

Without calculating B or A, find B – A.

CHAPTER 2

Integers & Operations

Target Concepts and Skills

- ☐ Negative and positive integers
- ☐ Integer operations
- ☐ Odd and even numbers
- ☐ Evaluating expressions
- ☐ Additive and multiplicative inverses
- ☐ Consecutive numbers and sums

Definition 2.1

Any naural number is positive. The set {1, 2, 3, 4, 5, ...} is called the set of positive integers.

Given a natural number n, we define its opposite −n as the unique number −n such that.

$n + (-n) = (-n) + n = 0.$

The collection {−1, −2, −3, −4, −5, ...} of all the opposites of the natural numbers is called the set of negative integers.

Example 2.1

We have, $(+19) + (-21) = -2$, since the loss of 21 is larger than the gain of 19 and so we obtain a loss.

Example 2.2

We have, $(-10) + (+21) = +11$, since the loss of 10 is smaller than the gain of 21 and so we obtain a gain.

Example 2.3

- We have, $(+8) - (+5) = (+8) + (-5) = 3$.
- We have, $(-8) - (-5) = (-8) + (+5) = -3$.
- We have, $(+8) - (-5) = (+8) + (+5) = 13$.
- We have, $(-8) - (+5) = (-8) + (-5) = -13$.

Remark 2.1

Order of Operations

1. Groupings (parenthesis)	$10^2 - (8 + 4) + 5 \times 2$
	$10^2 - 12 + 5 \times 2$
2. Exponents	$100 - 12 + 5 \times 2$
3. Multiplication and/or Division left → right	$100 - 12 + 10$
	$88 + 10$
4. Addition and/or Subtraction left → right	98

Problem 2.1

Evaluate: $\dfrac{16 - (4 - 2)}{14 \div (3 + 4)}$

Solution 2.1

$\dfrac{16 - (4 - 2)}{14 \div (3 + 4)}$ {parenthesis first}

$= \dfrac{16 - 2}{14 \div 7}$ {evaluate numerator, denominator}

$= \dfrac{14}{2}$ {do the division}

$= 7$

Problem 2.2

Evaluate $8 + 2^3 \div 4 - 5 + 3 \cdot 4$

Solution 2.2

$8 + 2^3 \div 4 - 5 + 3 \cdot 4$

$= 8 + 8 \div 4 - 5 + 3 \cdot 4$ (evaluate exponent)

$= 8 + 2 - 5 + 12$ (multiply and divide)

$= 10 - 5 + 12$ (add and subtract)

$= 5 + 12$

$= 17$

Remark 2.2

Negative bases-odd and even power

A negative base raised to an even power is positive

A negative base raised to an odd power is negative.

Example:

$(-1)^2 = -1 \times -1 = 1$

$(-1)^3 = -1 \times -1 \times -1 = -1$

$(-1)^4 = -1 \times -1 \times -1 \times -1 = 1$

$(-2)^2 = -2 \times -2 = 4$

$(-2)^3 = -2 \times -2 \times -2 = -8$

$(-2)^4 = -2 \times -2 \times -2 \times -2 = 16$

Problem 2.3

Evaluate

a. $(-3)^2$ c. $(-3)^3$ e. $-(-3)^3$

b. -3^2 d. $-(-3)^4$

Solution 2.3

a. $(-3)^2 = 9$

b. $-3^2 = -1 \times 3^2 = -9$

c. $(-3)^3 = -27$

d. $-(-3)^4 = -81$

e. $-(-3)^3 = -1 \times (-3)^3 = -1 \times -27 = 27$

Definition 2.2

A whole number is even if it has 2 as a factor and thus is divisible by 2.

A whole number is odd if it is not divisible by 2.

Remark 2.2

0 : Odd number E : Even number

Addition & Subtraction

±	O	E
O	E	O
E	O	E

Multiplication

x	O	E
O	O	E
E	E	E

Example 2.4

a. The sum of two even numbers is always even

b. The sum of two odd numbers is always even

c. The sum of an odd number and an even number is always odd

d. The sum of three odd numbers is always odd

Remark 2.3

Numbers of numbers

If a and b are natural numbers with a > b, then there are (a – b) – 1 natural numbers between a and b.

If a and b are natural numbers with a > b, then there are (a – b) +1 natural numbers between a and b (a and b are included)

Example 2.5

What is the sum

$1 + 2 + 3 + \cdots + 99 + 100$

of all the positive integers from 1 to 100?

We can pair up the numbers into the fifty pairs

$(100 + 1) = (99 + 2) = (98 + 3) = \cdots = (50 + 51)$.

Thus we have 50 pairs that add up to 101 and the sum is $50 \times 101 = 5050$.

Problem 2.4

Find the sum of

$2 + 4 + 6 + 8 + \ldots + 48 + 50$.

Solution 2.4

$2 + 4 + 6 + 8 + \ldots + 48 + 50$

$= (2\cdot 1) + (2\cdot 2) + (2\cdot 3) + \ldots + (2\cdot 24) + (2\cdot 25)$

$= 2\cdot(1 + 2 + 3 + \ldots + 24 + 25)$

$= \cancel{2}\cdot\dfrac{25\cdot(25+1)}{\cancel{2}} = 25\cdot 26 = 650.$

Remark 2.4

If x is the larger between x and y, the difference between x and y is x − y. However, if y is the larger between x and y, the difference between x and y is y − x.

Remark 2.5

If n is an integer, its predecessor is n −1 and its successor is n +1. So, the sum of three consecutive numbers is $n - 1 + n + n + 1 = 3n$.

Definition 2.3 – Additive and Multiplicative inverses

For each Real number of a, the equations $a + x = 0$ and $x + a = 0$ have a solution called the **additive inverse** of a, and denoted by –a.

For each Real number of a, **except for 0**, the equations $a \times x = 1$ and $x \times a = 1$ have a solution called the **multiplicative inverse** of a, and denoted by a^{-1} (and often written as 1/a or $\frac{1}{a}$).

Example 2.7

The additive inverse of 2/3 is –2/3.

The multiplicative inverse of 2/3 is 3/2.

Problem 2.5

If $a + \frac{8}{3} = 0$ and $b \times \frac{3}{2} = 1$,

What is $a + b = ?$

Solution 2.5

a is an additive inverse of $\frac{8}{3}$.

So, $a = -\frac{8}{3}$.

b is a multiplicative inverse of $\frac{3}{2}$.

So, $b = \frac{2}{3}$.

$a + b = -\frac{8}{3} + \frac{2}{3} = -\frac{6}{3} = 2.$

Definition 2.1 – Consecutive Numbers

Consecutive numbers are numbers that follow each other in order. They have a difference of 1 between every two numbers. If n is a number, then n, n + 1, and n + 2 would be consecutive numbers

Example 2.8

The consecutive odd integers between 1 and 15 are 1, 3, 5, 7, 9, 11, 13, and 15.

The consecutive odd integers between –11 and –1 are –11, –9, –7, –5, –3, and –1.

Remark 2.6

The sum of n consecutive numbers is divisible by n if n is an odd number. For example, let us consider a consecutive odd number sequence 5, 7, 9, 11, 13, 15, 17 which has 7 numbers in the sequence. So according to the property, the sum of this consecutive odd number sequence should be divisible by 7.

$5 + 7 + 9 + 11 + 13 + 15 + 17 = 77.$

$77 / 7 = 11$ which satisfies this property.

Problem 2.6

If $a < b < c < d$ and a, b, c, and d are consecutive odd numbers what is $(b – a)\ (d – a)$?

Solution 2.6

Let a be n then $b = a + b$, $c = a + 4$, and

$d = a + 2$, $c = a + 4$, and $d = a + 6$

$b – a = \cancel{a} + 2 – \cancel{a} = 2$, and $d – a = (\cancel{a} + 6) – (\cancel{a})$

So, $(b – c)(d – a) = 2 \cdot 6 = 12$

The First 4 to 25

Using each of the digits 1, 2, 3, and 4, once and only once, with the basic rules of arithmetic (+, –, ×, ÷, and parenthesis), express all of the integers from 1 to 25. For example, $1 = 2 \times 3 – (1 + 4)$

Answer	Solution 1	Solution 2
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		
25		

1. If $9210 – 9124 = 210 – \square$, the value represented by the $\square$ is

2. Operations are placed in each ○ so that
$3 ○ 5 ○ 7 ○ 9 = 78$.
Listed from left to right, the operations are

3. When two numbers are added, the result is –26. If one of the numbers is 11, what is the other number?

4. If $y = 3$, the value of $\dfrac{y^3 + y}{y^2 – y}$ is

5. In the addition shown, P and Q each represent single digits, and the sum is 1PP7.

$$\begin{array}{r} 7\ 7\ P \\ 6\ Q\ P \\ +\ \ Q\ Q\ P \\ \hline 1\ P\ P\ 7 \end{array}$$

What is P + Q?

6. If a, b and c are positive with
$a \times b = 13$, $b \times c = 52$, and $c \times a = 4$,
the value of $a \times b \times c$ is

7. If a is an even integer and b is an odd integer, which of the following could represent an odd integer?
A) ab B) $a + 2b$ C) $2a – 2b$
D) $a + b + 1$ E) $a – b$

8. In the addition of three-digit numbers shown, the letters x and y represent different digits.

$$\begin{array}{r} 3\ x\ y \\ +\ \ y\ x\ 3 \\ \hline 1\ x\ 1\ x \end{array}$$

What is $y – x$?

9. The sum of the first 100 positive integers (that is, $1 + 2 + 3 + ... + 99 + 100$) equal 5050.
The sum of the first 100 positive multiples (that is, $10 + 20 + 30 + ... + 990 + 1000$) equals.

10. The sum of three consecutive integers is 153. The largest of these three integers is

11. If $12 \times \square = 360$

What is $\square$?

12. How many different numbers can we create by replacing parenthesis to the expression

$4 \times 4 + 4 \times 4$?

13. I'm thinking of two whole numbers. Their product is 36 and their sum is 13. What is the larger number?

14. How many perfect squares are less than 1000?

15. What is the tens digits of the product

$5 \times 6 \times 7 \times 8 \times 9$?

16. $4^3 + 6^2 = 10^x$

What is x?

17. If $a < b < c < d$ and a, b, and c, are consecutive odd numbers what is $(c - b)(d - a)$?

18. $-3(5 - 6) - 4(2 - 3) = ?$

19. If x and y are integers and $x = \dfrac{15}{y}$.

How many distinct values of x can be found?

20. What number increases by 16 when tripled?

CHAPTER 3

Primes & Divisibility

Target Concepts and Skills

- ☐ Prime numbers & composite numbers
- ☐ Prime factorizations
- ☐ Divisors & Factors
- ☐ Perfect squares & perfect cubes
- ☐ Divisibility Rules

Definition 3.1

Primes and Composite Numbers

A prime number is a whole number greater than 1 that has exactly two factors, 1 and itself. Any number with more than two factors is a composite number. For example, 13 is prime since 1 and 13 are its only factors; 10 is composite since it has factors of 1, 2, 5, and 10.

Remark 3.1

The unique even prime number is 2. All other primes are odd primes.

Problem 3.1

Determine whether each number is prime or composite:

a. 20 b. 29 c. 121 d. 91

Solution 3.1

a. 20 is composite, because 20 can be written as $20 = 4 \times 5$ or 2×10.

b. 29 is prime, because 29 can only be factored as $29 = 1 \times 29$, which means the only positive divisors of 29 are 1 and 29.

c. 121 is composite, because $121 = 11 \times 11$.

d. 91 is composite, because $91 = 7 \times 13$

Remark 3.2 – Primes less than 100. The Sieve of Eratosthenes.

One method for finding the prime numbers is by using the Sieve of Eratosthenes.

Here are the steps to this algorithm, using the following table:

1. Cross out 1 (it is not prime)
2. Circle 2 (it is prime) and then cross out all multiples of 2
3. Circle 3 (it is prime) and then cross out all multiples of 3
4. Circle 5, then cross out all multiples of 5
5. Circle 7, then cross out all multiples of 7
6. Continue by circling the next number not crossed out, then cross out all of its multiples

The circled numbers are all the prime numbers less than 100.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Example 4.4

If we know the prime factorizations of 40 and 60, we can just write them and the common factors.

$40 = 2 \cdot 2 \cdot 2 \cdot 5$

$60 = 2 \cdot 2 \cdot 3 \cdot 5$

Multiply the circled numbers: $2 \cdot 2 \cdot 5 = 20$.

Therefore, the greatest common factor of 40 and 60 is 20. We can write GCF(40, 60) = 20.

Definition 4.4 – The Least Common Multiple (LCM)

A multiple of a number is the product of that number and any other whole number.

The least common multiple (LCM) of two or more natural numbers is the smallest element in the set of common multiples of the numbers. We write LCM(a, b) to mean the least common multiple of a and b.

Example 4.5

36 is a multiple of 4 since $4 \times 9 = 36$.

In looking for a multiple of a number, we usually begin with the number and generate multiples of it. The positive multiples of 4 are 4, 8, 12, 16, 20, 24, 28, ...

Example 4.6

For example, let us find the least common multiple of 6 and 8.

The multiples of 6 are

{6, 12, 18, 24, 30, 36, 42, 48, ...}.

The multiples of 8 are

{8, 16, 24, 32, 40, 48, 56 ...}.

The common multiples of 6 and 8 are {24, 48, ...}.

Therefore, the least common multiple of 6 and 8 is 24. We can write LCM(6, 8) = 24.

Example 4.7

For example, let us LCM(10, 12).

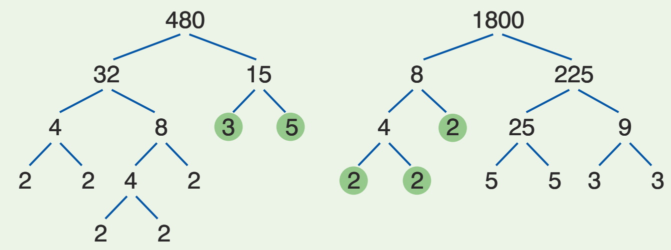
Look at the result of the division method. The prime factors are 2, 2, 3, and 5. Therefore,

$LCM(10, 12) = 2 \cdot 2 \cdot 3 \cdot 5 = 60$.

10	12	2
5	6	2
5	3	3
5	1	5
1		

Example 4.8

Find the GCD of 480 and 1800



$GCD(480, 1800) = 2 \times 2 \times 2 \times 3 \times 5 = 120$

Remark 4.1

LCM (m, m + 1) for any positive integer m is the product of m and m + 1.

Remark 4.2

GCD (m, m + 1) for any positive integer is m is 1.

Remark 4.3

Let a and b be two natural numbers, then

$GCD(a, b) \cdot LCM(a, b) = a \cdot b$.

Problem 4.1

Find n if GCD(n, 18) = 6 and LCM(n, 18) = 36

Solution 4.1

Using the property, $GCD(n, 18) = n \cdot 18$.

Therefore, $6 \cdot 36 = n \cdot 18$, and

so $n = \dfrac{6 \cdot 36}{18} = 6 \cdot 2 = 12$

MathTopia Press

1. All numbers divisible by both 15 and 12 are also divisible by which of the following?

A) 35 B) 41 C) 30 D) 41 E) 60

2. All numbers divisible by both 14 and 10 are also divisible by which of the following?

A) 10 B) 42 C) 50 D) 28

3. Find the GCD and LCM of 24 and 40. Then multiply the GCD by the LCM. What do you notice?

4. How many distinct prime factors does 60 have?

5. LCM(12, 18, 24) = ?

6. GCD(72, 80) = ?

7. What is the least common multiple of 24, 30, 36?

8. Find the least common multiple of 4, 12, and 18.

9. Find the greatest common factor of 32, 48, and 64.

10. Let A, B, and a be natural number with

A = GCD(a, 12) and B = LCM(a, 12).

If $A \cdot B = 216$, find a.

MathTopia Press

Make the number 24 from the four numbers shown. You can add, subtract, multiply and divide. Use all four numbers on the card, but use each number only once. You do not have to use all four operations.

1. a)

b)

c)

2. a)

b)

c)

3. a)

b)

c)

4. a)

b)

c)

1. Which one is closest to 0?

a) 1/2 b) 1/8 c) 1/3 d) 1/9

2. If $\frac{1}{15} = \frac{x}{60} = \frac{y}{30}$

What is x + y?

3. Which one's reciprocal is bigger?

A) –2 B) –1

4. $(6 \div -3)(4 - 12) = ?$

5. Which one is bigger?

A) $\frac{1}{2} \times \frac{1}{4}$ B) $\frac{1}{2} \div \frac{1}{4}$

6. Which one is smaller?

A) $\frac{1}{3} - \frac{1}{5}$ B) $\frac{1}{4} - \frac{1}{5}$

7. $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} = ?$

8. $4\frac{3}{45} + 1\frac{4}{15} = ?$

9. What is x?

$\frac{3}{7} = \frac{x}{63}$

10. If $\frac{3}{4} + \frac{4}{x} = 1$

What is x?