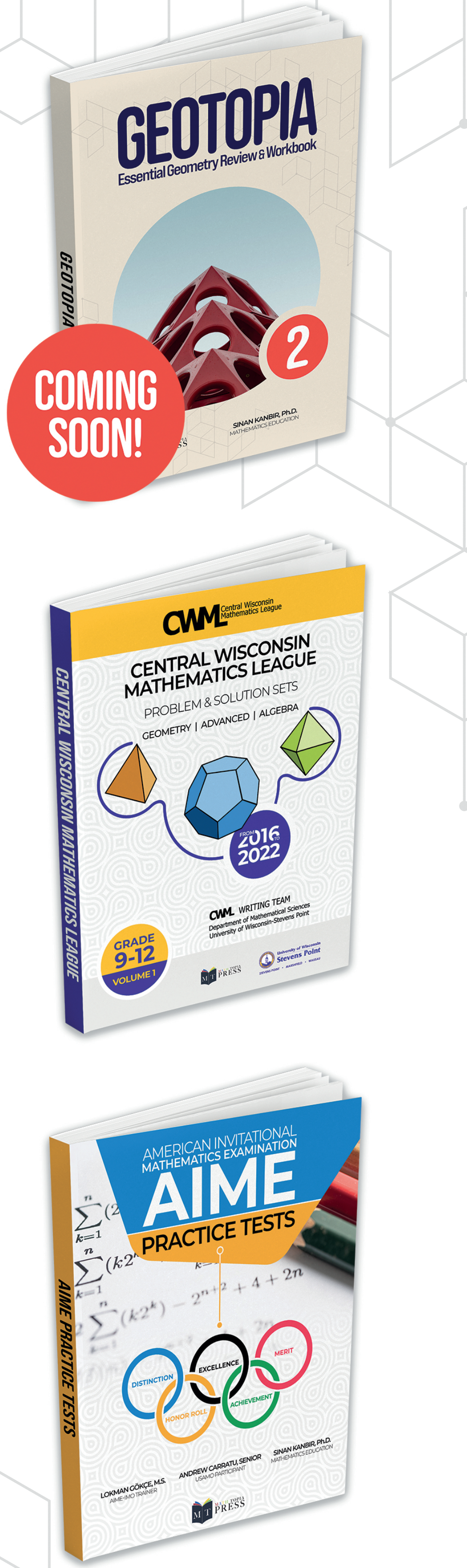
MATHTOPIA PRESS | GRADES 6–12

Competition-aligned Math With Curriculum Support



About the Author — Sinan Kanbir, PhD

Sinan Kanbir is a math education professor who loves helping students understand and enjoy geometry. He teaches future teachers at the **University of Wisconsin–Stevens Point** and creates math materials for **K–12 students**.

Dr. Kanbir created the **GeoTopia** courses and teaches **GeoTopia** summer camps, where middle and high school students explore geometry through visuals, patterns, and meaningful problem solving. Instead of focusing on memorization, his approach helps students learn how to see structure, think logically, and explain their ideas clearly.

He also helps write math contest problems for organizations such as the **Central Wisconsin Math League (CWML), MathCON**, and the **MAA Wisconsin Section Math Contest.** Dr. Kanbir believes that everyone can learn geometry and that math can be challenging, creative, and fun.

Welcome to GeoTopia 0.5

A Message to Students

Why Start Geometry Early?

In the United States, geometry is often taught in **high school (Grades 9 or 10).** However, students can begin learning geometry as early as **7th grade**.

Starting earlier helps you:

- Improve **spatial reasoning**
- Strengthen **visual thinking**
- Build a solid foundation in **Euclidean geometry**

GeoTopia 0.5 is designed to help you grow these skills step by step—before formal proofs begin.

How This Book Helps You Learn

By working through the **GeoTopia 0.5** problems, you will:

- Learn to **see patterns and structure** in shapes
- Practice explaining ideas using **clear mathematical reasoning**
- Strengthen your **algebra skills** by working with formulas, variables, and relationships
- Improve your **math reading skills** by following **short, logical solutions**

Geometry is not just about shapes—it helps you think more clearly in **all areas of math**.

✏️ How to Use This Book Successfully

- Draw diagrams
- Take your time
- Make mistakes and try again
- Read the **short solutions** carefully
- Ask yourself **why** each step works

Every problem is an opportunity to grow your thinking.

💡 Remember

Geometry is not about memorizing formulas.

It is about **seeing, thinking**, and **making sense of ideas**.

If you stay curious and keep practicing, you will build confidence and discover that geometry can be **logical, creative, and enjoyable**.

What is GeoTopia 0.5 and What Makes This Book Special and Unique?

GeoTopia 0.5: Essential Geometry Review and Workbook is a comprehensive **bridge-level geometry resource** designed to strengthen students' core geometric understanding while preparing them for **school assessments, standardized exams, and math competitions**. Positioned between **GeoTopia 0** and **GeoTopia 1**, this book builds the essential foundations needed for advanced reasoning, formal geometry, and contest-level thinking.

GeoTopia 0.5 is **fully aligned with the New York State Regents Geometry Exam, fits 100% of most states' end-of-year geometry exams**, and provides strong preparation for **AMC 8 and AMC 10 geometry topics**, as well as the **SAT and ACT geometry sections**. The book intentionally connects **classroom geometry, standardized test preparation**, and **competition-style problem solving** through a **balanced, visual-first approac**h.

Rather than emphasizing memorization, GeoTopia 0.5 helps students develop **geometric intuition, logical structure, and confidence** in applying ideas to unfamiliar problems. Visual reasoning, pattern recognition, and short, clear solutions make abstract concepts accessible and meaningful.

Inside, students will find **1000+ carefully sequenced problems, step-by-step worked examples**, and **visual proofs** that support both **conceptual understanding and procedural fluency**. Each chapter follows a structured progression that guides students from learning and exploration to reasoning and application, ensuring deep and lasting understanding.

Whether used for **self-study, classroom instruction, enrichment programs, gifted education, summer camps, or competition preparation**, GeoTopia 0.5 prepares learners not only to succeed on exams—but to **think geometrically with clarity and confidence**.

Target Audiences of This Book

GeoTopia 0.5 is primarily designed for students in **Grades 8–10**, with content and pacing aligned to middle and early high school geometry expectations. It is also well suited for **advanced or accelerated 7th-grade students** who are ready to begin geometry earlier.

This book is ideal for:

- **Students in Grades 8–10** studying or preparing for formal geometry
- **Advanced and accelerated 7th graders** with strong mathematical readiness
- **Students preparing for the New York State Regents Geometry Exam** and **most states' end-of-year geometry exams**
- **Students preparing for standardized tests**, including the **SAT** and **ACT** geometry sections
- **Students preparing for math competitions**, especially **AMC 8** and **AMC 10** geometry topics
- **Students transitioning into high school geometry** who need a strong conceptual foundation
- **Advanced and gifted learners** seeking deeper reasoning and challenge
- **Math circles, enrichment programs, and summer camps**
- **Classroom teachers and homeschool educators** seeking a comprehensive and flexible geometry resource
- **Independent learners** who benefit from clear explanations and worked examples

Whether used for **classroom instruction, enrichment, self-study, or competition preparation**, GeoTopia 0.5 supports students in developing **strong geometric reasoning, fluency, and confidence**.

🎯 Competition & Exam Alignment – GeoTopia 0.5

Math Competitions

Competition	Alignment
AMC 8 (Geometry)	✓ 100% aligned
AMC 10 (Geometry)	✓ 100% aligned
MathCON (Grades 6-10)	✓ Full alignment
MathCounts (Geometry enrichment)	✓ Full alignment

Standardized & State Exams

Exam	Alignment
New York State Regents Geometry Exam	✓ 100% aligned
State End-of-Year Geometry Exams	✓ Strong alignment
SAT (Geometry)	✓ 100% aligned
ACT (Geometry)	✓ 100% aligned

📘 When to Choose GeoTopia 0.5

GeoTopia 0.5 is ideal for students who:

- Are preparing for **AMC 8 or AMC 10 geometry** with full topic coverage
- Are beginning or strengthening **high school geometry**
- Need a **conceptual, visual-first bridge** before formal proofs
- Are preparing for **Regents, SAT, and ACT** exams

How GeoTopia 0.5 Aligns with Exams & Math Competitions

GeoTopia 0.5 is a bridge-level geometry workbook designed to build v*isual reasoning, diagram interpretation, and conceptual fluency*. Its worked examples and carefully sequenced problem sets align with the structure, rigor, and expectations of major U.S. state exams and national math competitions.

NYSED Geometry Regents Exam

A comprehensive state exam administered by the New York State Education Department. It emphasizes accurate diagram use, logical explanations, and precise application of geometric properties involving angles, triangles, parallel lines, and polygons. GeoTopia 0.5 mirrors Regents-style expectations through concise explanations and step-by-step reasoning. Official site: nysed.gov/state-assessment/geometry

High School Geometry State Exams (General Passing Exams)

End-of-course or graduation-required geometry exams used across many U.S. states. These exams assess mastery of angles, triangles, polygons, circles, similarity, congruence, and coordinate geometry.

GeoTopia 0.5 supports these exams by reinforcing core concepts, diagram interpretation, and structured reasoning commonly required for passing.

Representative state exams include:

- **Texas STAAR (Geometry / EOC)** — emphasizes geometric relationships, transformations, and problem-solving using diagrams.

Official site: tea.texas.gov/student-assessment/staar

- **California State Geometry Exams (CAASPP / Smarter Balanced)** — focuses on conceptual understanding, visual modeling, and application of geometry standards.

Official site: cde.ca.gov/ta/tg/ca

AMC 8

A national middle-school competition focused on mathematical reasoning rather than advanced algebra. Geometry problems rely heavily on visual insight, angle chasing, symmetry, and spatial reasoning. GeoTopia 0.5 develops these skills through diagram-driven worked examples and targeted practice.

Official site: maa.org/student-programs/amc

MathCounts

A rigorous middle-school competition requiring creative, non-routine problem-solving under time pressure. Geometry problems often combine multiple concepts within a single diagram. GeoTopia 0.5 builds flexible thinking and efficient geometric strategies essential for MathCounts success.

Official site: mathcounts.org

MathCON

A concept-based competition for Grades 4–8 emphasizing understanding over memorization. Geometry problems test spatial reasoning, interpretation, and logical consistency. GeoTopia 0.5 aligns closely through accessible explanations and concept-focused problem sets.

Official site: mathcon.org

SAT

A college entrance exam assessing applied geometry in real-world and abstract contexts. Students must interpret diagrams, connect multiple concepts, and reason efficiently. GeoTopia 0.5 reinforces the geometric fluency and diagram interpretation skills needed for SAT preparation.

Official site: collegeboard.org

ACT

A fast-paced college readiness exam where geometry questions reward quick recognition of standard configurations and efficient solution paths. GeoTopia 0.5 trains students to solve accurately using minimal, well-organized steps.

Official site: act.org

Summary

Across state exams and national competitions, **GeoTopia 0.5** develops strong diagram literacy, conceptual geometric understanding, and efficient, exam-ready problem-solving skills suitable for middle school through early high school learners.

Acknowledgments

This book was shaped by a shared love of mathematics and the joy of deep thinking. The author gratefully acknowledges the exceptional problem-solving insight, thoughtful feedback, and careful review of the following students:

Holly Speranza (Rhode Island)

Itai Firstenberg (California)

Ayaan Garg (Massachusetts)

Aisha Tanimu (Wisconsin)

Furkan Akif Doğan (Texas)

Yusuf Oktay (North Carolina)

Their creativity, precision, and commitment to mathematical clarity meaningfully strengthened this book.

The author also extends sincere thanks to the MathTopia Academy instructors—Mr. Annayev, Andrew Carratu (MIT), Sarah Zuge (Georgia Institute of Technology), and Ava Kumar (Cornell University)—for their mentorship, guidance, and continued inspiration.

Finally, heartfelt appreciation goes to the students from MathPath, MathCON, and the MathTopia Academy Summer Math Camps, whose curiosity, persistence, and enthusiasm for problem-solving helped shape the vision and development of GeoTopia 0.5. Special thanks are extended to long-time problem solvers and dedicated volunteers, including Cindy Speranza (Rhode Island) and Maitreya Mukhopadhyay (Texas), whose continued commitment to supporting mathematically promising students—and whose encouragement throughout the process—made this work possible.

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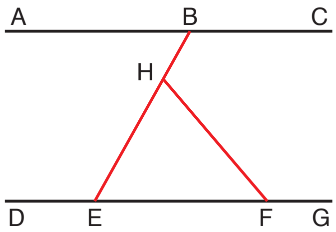
SOLUTIONS

GeoTopia 0.5 Pre-Test & Evaluation

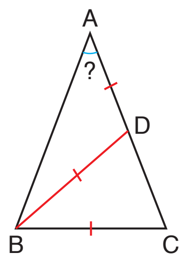
1. **(NYSED, Geometry Regents Exam, Jan 16 Q. 12)**

In the diagram below, $\overline{ABC} \parallel \overline{DEFG}$. Transversal $\overline{BHE}$ and line segment HF are drawn.



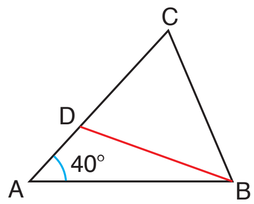
If m∠HFG = 130° and m∠EHF = 70°, what is m∠ABE?

2. Triangle ABC is an isosceles triangle with AB = AC and AD = BD = BC. What is the measure of the angle BAC?



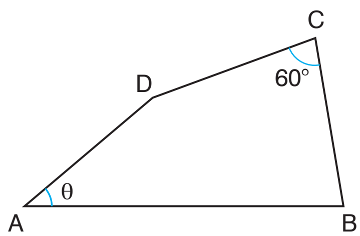
3. In the diagram, AB = AC and D is a point on AC such that BD = BC. Angle BAC is 40°.

What is angle ABD?



4. BC = CD = DA, ∠C = 60°, and m∠ADC = 150°

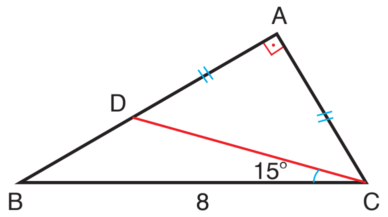
m∠A = ?



5. In right triangle ABC

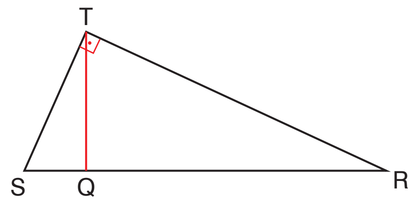
- $AB \perp AC$.
- $\angle BCD = 15°$.
- $AD = AC$
- $BC = 8$ cm.

Find the length of AB.



6. **(NYSED, Geometry Regents Exam, Aug 23 Q. 30)**

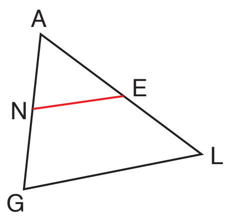
Right triangle STR is shown below, with $m\angle T = 90°$. Altitude $\overline{TQ}$ is drawn to $\overline{SQR}$, and $TQ = 8$.



If the ratio SQ : QR is 1:4, determine and state the length. of $\overline{SR}$.

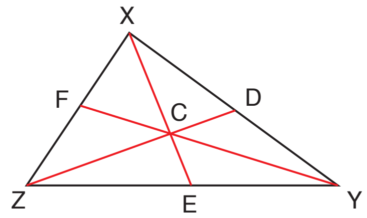
7. **(NYSED, Geometry Regents Exam, Aug 24 Q. 29)**

In ΔAGL below, N and E are the midpoints of $\overline{AG}$ and $\overline{AL}$,respectively.



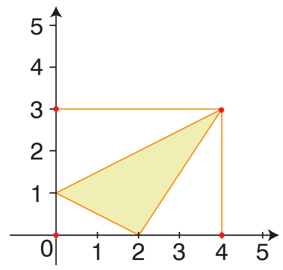
If $NE = 15$ and $GL = 3x - 12$, determine and state the value of x.

8. In ΔXYZ shown below, medians $\overline{XE}$, $\overline{YF}$, and $\overline{ZD}$ intersect at C.



If CE= 5, YF = 21, and XZ = 15, determine and state the perimeter of triangle CFX.

9.



What is the area of the shaded triangle?

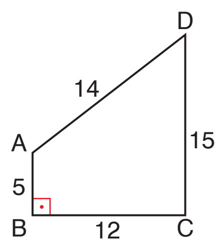
10. In the figure,

AB = 5 cm,BC = 12 cm, CD =15 cm, AD = 14 cm, with $\angle ABC = 90°$.

The area of a triangle with sides a, b, c is given by Heron's formula:

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}, \text{where } s = \frac{a+b+c}{2}.$$

Using this, find the area of quadrilateral ABCD.



My Notes:

My Notes:

GeoTopia 0.5 Pre-Test & Evaluation

Problem	Topic	My Answer	Correct Answer	Notes
1	Angles (Problem 1)			
2	Angles (Problem 2)			
3	Triangles: Angles & Inequalities (Problem 1)			
4	Triangles: Angles & Inequalities (Problem 2)			
5	Triangles: Right & Special (Problem 1)			
6	Triangles: Right & Special (Problem 2)			
7	Triangles: Lengths (Problem 1)			
8	Triangles: Lengths (Problem 2)			
9	Triangles: Areas (Problem 1)			
10	Triangles: Areas (Problem 2)			
11	Polygons: Angles (Problem 1)			
12	Polygons: Angles (Problem 2)			
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15	Polygons: Areas (Problem 1)			
16	Polygons: Areas (Problem 2)			
17	Circles: Angles (Problem 1)			
18	Circles: Angles (Problem 2)			
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23	Analytical Geometry (Problem 1)			
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25	Three-Dimensional Geometry (Problem 1)			
26	Three-Dimensional Geometry (Problem 2)			
27	Trigonometry (Problem 1)			
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29	Transformations (Problem 1)			
30	Transformations (Problem 2)			
31	Miscellaneous (Problem 1)			
32	Miscellaneous (Problem 2)			

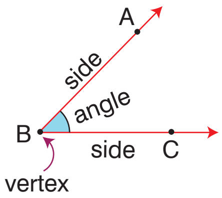
CHAPTER 1

Angles

Definition 1.1

Angle, Side, and Vertex

An **angle** is a union of two rays that have a common endpoint. The two rays are called the **sides** of the angle. The common endpoint is called the **vertex** of the angle.



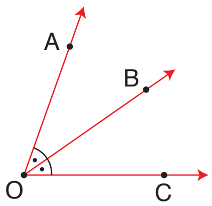
Definition 1.2

Angle Bisector

An **angle bisector** is a line ray or line segment that divides an angle into two congruent angles. In the figure, ray OB is a bisector of ∠AOC because

m∠AOB = m∠BOC and

m∠AOB = $\frac{1}{2}$ m∠AOC.

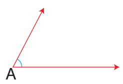


Definition 1.3

Types of Angles (by Measure)

- **Acute Angle:**

  An angle ∠A is **acute** if 0° < m∠A < 90°.

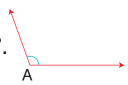


- **Right Angle:**

  An angle ∠A is **right angle** if m∠A = 90°.

- **Obtuse Angle:**

  An angle ∠A is **obtuse** if 90° < m∠A < 180°.

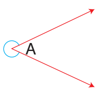


- **Straight Angle:**

  An angle ∠A is **straight** if m∠A = 180°.

- **Reflex Angle:**

  An angle ∠A is **reflex** if 180° < m∠A < 360°.



- **Complete Angle:**

  An angle ∠A is **complete** if m∠A < 360°.

Definition 1.4

Complementary Angles

Two angles are called complementary if the sum of their measure is 90°.

m∠A + m∠B = 90°

Each angle is called the **complement** of the other.

Definition 1.5

Supplementary Angles

Two angles are called supplementary if the sum of their measures is 180°.

m∠A + m∠B = 180°

Each angle is called the **supplement** of the other.

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Example 1.1

∠ABC is a straight angle.

If ∠ABD = 2y + 14 and ∠DBC = y + 22,

find y and each angle.

Solution:

(2y + 14) + (y + 22) = 180

3y + 36 = 180 ⇒ y = 48

∠ABD = 2(48) + 14 = 110°, ∠DBC = 48 + 22 = 70°

Example 1.2

Two angles are complementary. One is x^0, the other is (3x – 10)°. Find x.

Solution:

x + (3x – 10) = 90

4x – 10 = 90 ⇒ 4x = 100 ⇒ x = 25

Angles: 25° and 65°

Example 1.3

In ABC, $\overline{BD}$ bisects ∠ABC. If ∠ABD = 2x + 5 and ∠DBC = x + 17, find x.

Solution:

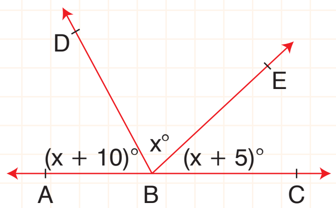
Since BD bisects ∠ABC:

2x + 5 = x + 17

x = 12

∠ABD = ∠DBC = 29°, ∠ABC = 58°

Example 1.4



Points A, B and C area on the same line. What is the measure of angle ∠ABE?

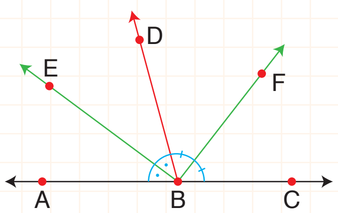
Solution:

(x + 10) + x (x + 5) = 180 ⇒ 3x + 15 = 180 ⇒ x = 55.

m∠ABE = (x + 10) + x = 2x + 10 = 120°.

Example 1.5

In the figure, BE and BD are the bisectors of ∠ABD and ∠CBD, respectively. Find m∠EBF.



Solution:

Since A, B, C are collinear,

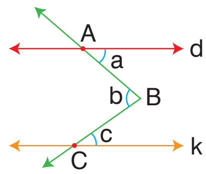
m∠ABD + m∠CBD = 180°

Because BE and BF are bisectors:

$m\angle EBF = \frac{1}{2}\, m\angle ABD + \frac{1}{2}\, m\angle CBD$

$m\angle EBF = \frac{1}{2}(180°) = 90°$

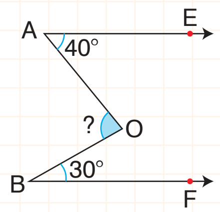
Property 1.1



In the figure, if d // k and B is the intersection of [BA and [BC, then

b° = a° + c°

Example 1.6



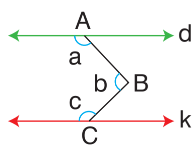
In the figure, [AE // [BF, m∠A = 40°, and m∠B = 30°. Find m∠AOB.

Solution:

m∠AOB = m∠OAE + m∠OBF

m∠AOB = 40° + 30 ° = 70°

Property 1.2



In the figure, if d // k and B is the intersection of [BA and [BC, then

a° + b° + c° = 360°.

CHAPTER 5

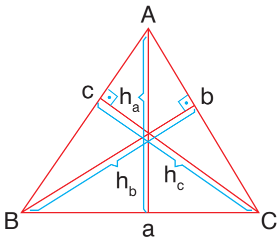
Triangles: Areas

Theorem 1.1

Acute Triangle's Area

Suppose a, b, and c are the length of sides and h_a, h_b, and h_c are length of altitudes of a, b, and c.

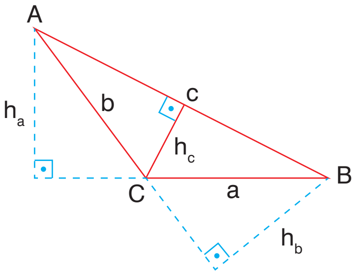
$$\text{Area} = \frac{a \cdot h_a}{2} = \frac{b \cdot h_b}{2} = \frac{c \cdot h_c}{2}$$



Theorem 1.2

Obsute Triangle's Area

$$\text{Area} = \frac{a \cdot h_a}{2} = \frac{b \cdot h_b}{2} = \frac{c \cdot h_c}{2}$$



Example 1.1

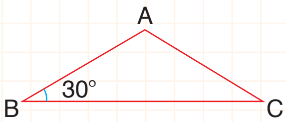
Suppose AB = 4 units, BC = 8 unit, and m∠B = 30°.

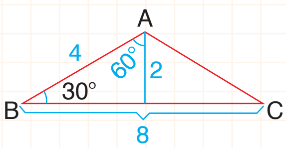
What is the area of ΔABC is square units?

Solution:

From 30° – 60° – 90° triangle, the height of ΔABC is 2 units.

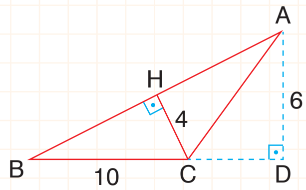
Area ABC $= \frac{8 \cdot 2}{2} = 8$ square units.





Example 1.2

In the figure, CH = 4, AD = 6, and BC = 10. What is AB?



Solution:

Use $A_\triangle = \frac{1}{2}bh$ with two different bases.

- With base BC = 10, the corresponding altitude is 6:

$$[ABC] = \frac{1}{2}(10)(6) = 30.$$

- With base AB, the corresponding altitude is 4:

$$[ABC] = \frac{1}{2}(AB)(4) = 2AB.$$

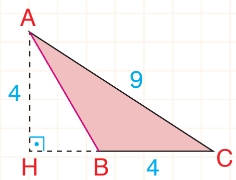
Equate the two expressions for the same area:

$$30 = 2AB \Rightarrow AB = 15.$$

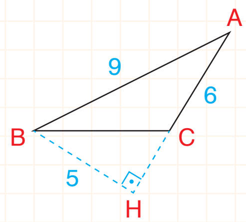
Example 1.3

Find the area of the triangle.

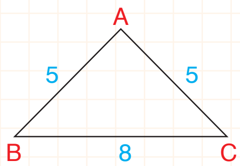
a.



b.



c.



Solution:

a. $A(ABC) = \frac{|BC| \cdot |AH|}{2} = \frac{4 \cdot 4}{2} = 8 \text{ cm}^2$

b. $A(ABC) = \frac{|AC| \cdot |BH|}{2} = \frac{6 \cdot 5}{2} = 15 \text{ cm}^2$

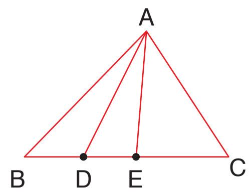
c. $|AH|^2 + 4^2 = 5^2 \Rightarrow |AH|^2 = 9$

$|AH| = 3 \Rightarrow A(ABC) = \frac{|BC| \cdot |AH|}{2} = \frac{8 \cdot 3}{2} = 12 \text{ cm}^2$

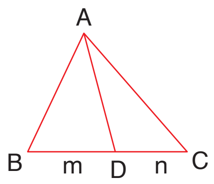
Quick Note

a. If two triangles have equal heighy, then ratio of their areas is the ratio of theis bases.

b. If two triangles have equal bases, then ratio of their areas is the ratio of their height.



Area(ABD): Area(ADE): Area(AEC) = BD : DE : EC

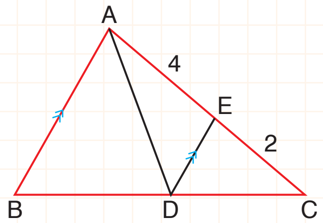


$\dfrac{\text{Area(ABD)}}{\text{Area(ADC)}} = \dfrac{m}{n}$

Example 1.4

Segments AB and ED are parallel. Area(ADE) = 12.

What is area(ABD)?



Solution:

Reason (using the area-base idea):

- ED ∥ AB ⇒ triangles ΔADE and ΔABD have the same height (distance from A to the parallel lines).

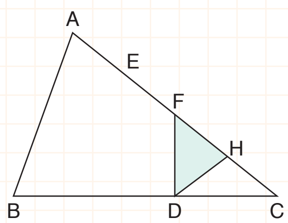
  Hence $\dfrac{[ADE]}{[ABD]} = \dfrac{DE}{AB}$.
- Because ED ∥ AB, $\Delta DEC \sim \Delta ABC$ (AA).

  Corresponding sides give $\dfrac{DE}{AB} = \dfrac{EC}{AC} = \dfrac{2}{4+2} = \dfrac{1}{3}$.
- From (1) and (2): [ADE] : [ABD] = 1 : 3.

  Given [ADE] = 12, we get [ABD] = 3 · 12 = 36.

Example 1.5

In triangle ABC, |AE| = |EF| = |FH| = |HC|, 2|BD| = 3 · |DC| and A(FDH) = 4 cm² then find A(ABC).



Solution:

1. Points A, E, F, H, C splint AC into four equal pieces ⇒ $\dfrac{FH}{AC} = \dfrac{1}{4}$.

2. ΔFDH and ΔADC have the same height from D to line AC (perpendicular FD).

So

$$\frac{[\triangle FDH]}{[\triangle ADC]} = \frac{FH}{AC} = \frac{1}{4} \Rightarrow [\triangle ADC] = 4 \cdot [\triangle FDH] = 4 \cdot 4 = 16.$$

3. On base BC, 2|BD| = 3|DC| ⇒ BD : DC = 3 : 2.

ΔABD and ΔADC share the same height from A to BC.

Hence

$$\frac{[\triangle ABD]}{[\triangle ADC]} = \frac{BD}{DC} = \frac{3}{2} \Rightarrow [\triangle ABD] = \frac{3}{2} \cdot 16 = 24.$$

4. Total area:

$[\Delta ABC] = [\Delta ABD] + [\Delta ADC] = 24 + 16 = 40$ cm²

Quick Note

Complete the Retangle

We can find the **area of a triangle using coordinates** - even if we don't know its side lengths!

Let's use the verices: A(2, 3), B(0, 0), C(4,1)

Step 1. Draw and Enclose

Plot the triangle and draw the **smallest rectangle** that completely encloses it.

The rectangle's area: $A_{rectangle} = 4 \times 3 = 12$

Step 2. Subtract Outside Triangles

There are **three outer triangles** labeled ①, ②, and ③.

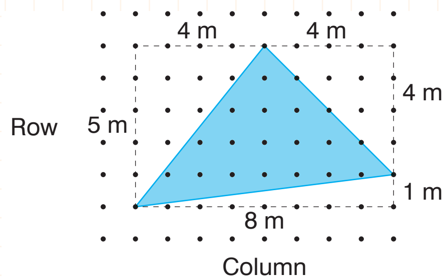
$$A_1 = \frac{2 \times 3}{2} = 3,\ A_2 = \frac{2 \times 2}{2} = 2,\ A_3 = \frac{4 \times 1}{2} = 2$$

Step 3. Find the Area of ΔABC

$A_{\Delta ABC} = A_{rectangle} - (A1 + A2 + A3 = 12 - (3 + 2 + 2) =$ 5 square units

Example 1.6

Alain has a 7 m × 9 m backyard and decides to construct a triangular-shaped patio. He plots his backyard plan on a graph as follows:



What is the area of his patio? How much space dose he have left in his backyard?

Solution:

$A_{patio} = A_{rectangle} - A_{①} - A_{②} - A_{③}$

$= (5 \times 8) - \frac{5 \times 4}{2} - \frac{4 \times 4}{2} - \frac{8 \times 1}{2} = 40 - 10 - 8 - 4 = 18\ m^2$

The area of Alain's patio is $18\ m^2$.

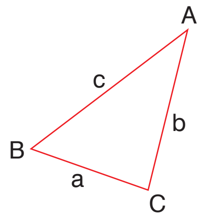
After building his patio, he will have $(7 \times 9) - 18 = 63 - 18$ $= 45\ m^2$ of his backyard left.

Theorem 1.3

Heron's Formula

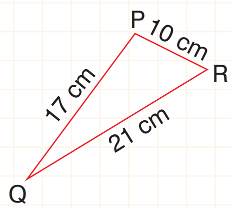
Let a, b, and c be the side engths of ΔABC. Then,

$A_{\Delta ABC} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a + b + c)$ is the semi-perimeter of ΔABC.



Example 1.7

Find the area of ΔPQR:



Solution:

$s = \frac{1}{2}(10 + 21 + 17) = 24$

$A_{\Delta PQR}$

$= \sqrt{24(24-10)(24-21)(24-17)}$

$= \sqrt{24(14)(3)(7)} = \sqrt{7056} = A_{\Delta PQR} = 84\ cm^2$

Practice 1

In rectangle ABCD, AB = 6 and AD = 8. Point M is the midpoint of AD. What is the area of ΔAMC?

A) 12 B) 15 C) 18 D) 20 E) 24

Solution:

$AB = 6$, $AD = 8$, M is midpoint of AD $\Rightarrow AM = \frac{AD}{2} = 4$.

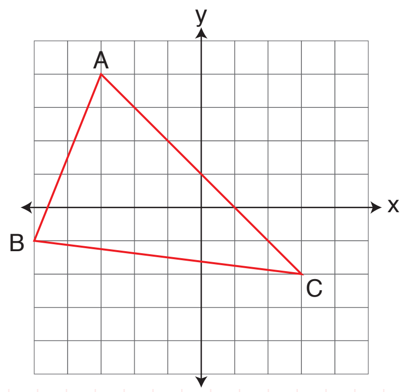
Line AM is a vertical side of the rectangle; the perpendicular distance from C to AM is the rectangle's width AB = 6.

Thus

$[\Delta AMC] = \frac{1}{2} \cdot (\text{base } AM) \cdot (\text{height} = 6) = \frac{1}{2} \cdot 4 \cdot 6 = 12.$

Practice 2

Triangle ABC is graphed on the set of axes below. The vertices of ΔABC have coordinates A(–3, 4), B(–5, –1) and C(3, –2).



What is the area of ΔABC?

Solution:

Complete-the-rectangle solution

A(–3, 4), B(–5, –1), C(3, –2) is $[x : -5 \to 3]$, $[y : -2 \to 4]$, so

$A_{rect} = (3 - (-5)) \cdot (4 - (-2)) = 8 \cdot 6 = 48.$

Outside of ΔABC there are three right triangles:

- **Top-left:** legs 2 and 5 $\Rightarrow$ area $\frac{1}{2} \cdot 2 \cdot 5 = 5.$
- **Top-right:** legs 6 and 6 $\Rightarrow$ area $\frac{1}{2} \cdot 6 \cdot 6 = 18.$
- **Bottom-left:** legs 8 and 1 $\Rightarrow$ area $\frac{1}{2} \cdot 8 \cdot 1 = 4.$

Subtract: $[\Delta ABC] = 48 - (5 + 18 + 4) = 21.$

Practice 3

If ΔABC is an isosceles triangle with the base $|BC| = a$. The midpoint of BC and AC are D and E respectively and $|AD| = |DE|$. Find the area of ΔABC?

Solution:

- In isosceles ΔABC with base BC = a, the midpoint D of BC makes AD a median and altitude $\Rightarrow AD \perp BC$.

  Let AD = h and $BD = \frac{a}{2}$.
- E is midpoint of AC. So DE is a midsegment in ΔABC: $DE \parallel AB$ and $DE = \frac{1}{2}AB$.
- Given $AD = DE \Rightarrow h = \frac{1}{2}AB \Rightarrow AB = 2h$.
- Right ΔABD : $AB^2 = AD^2 + BD^2 \Rightarrow (2h)^2 = h^2 + \left(\frac{a}{2}\right)^2$

  $\Rightarrow 3h^2 = \frac{a^2}{4} \Rightarrow h^2 = \frac{a^2}{12}$, so $h = \frac{a}{2\sqrt{3}}$.

Area: $[\Delta ABC] = \frac{1}{2} \cdot BC \cdot AD = \frac{1}{2} \cdot a \cdot \frac{a}{2\sqrt{3}} = \frac{a^2\sqrt{3}}{12}$

Practice 4

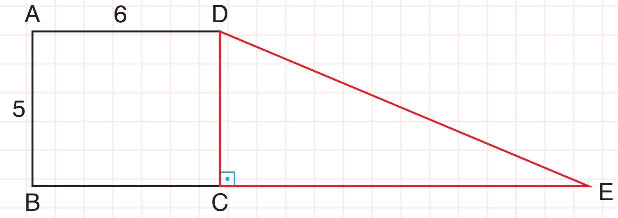
A triangle with vertices as A = (1, 3), B = (5, 1), and C = (4, 4) is plotted on a 6 × 5 gird. What fraction of the grid is covered by the triangle?

A) $\frac{1}{6}$ B) $\frac{1}{5}$ C) $\frac{1}{4}$ D) $\frac{1}{3}$ E) $\frac{1}{2}$

Solution:

Practice 6

Rectangle ABCD and right triangle DCE have the same area. They are joined to form a trapezoid, as shown. What is DE?



A) 12 B) 13 C) 14 D) 15 E) 16

Solution:

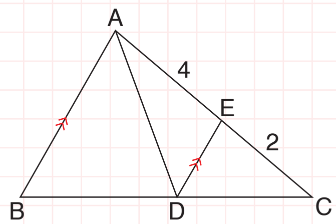
Practice 5

In ΔABC, AB = BC = 29, and AC = 42. What is the area of ΔABC?

A) 100 B) 420 C) 500 D) 609 E) 701

Solution:

Practice 7

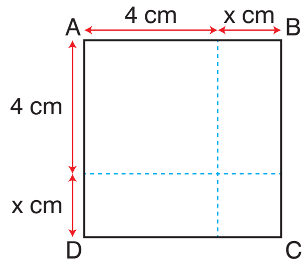


Segments AB and ED are parallel.

If the area of ΔEDC is 6, find area of ΔABD.

Solution:

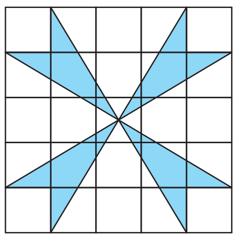
1.



The area of square ABCD is 24 cm².

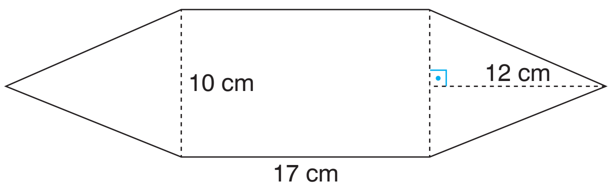
What is the valve of $x^2 + 8x$?

2. **(2007 AMC 8, #23)**



What is the area of the shaded pinwheel shown in the 5 x 5 grid?

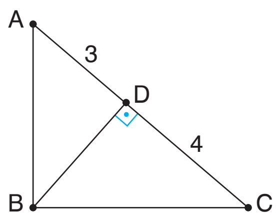
3. A banner is composed of two congruent triangles and a rectangle, as shown below.



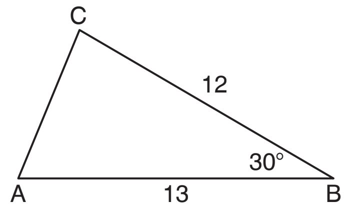
What is the total area of the banner in square centimeters?

4. **(2009 AMC, #10)**

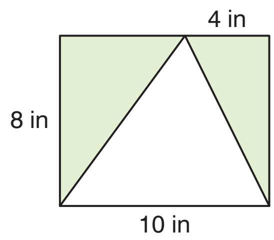
Triangle ABC has a right angle at B. Point D is the foot of the altitude from B, AD = 3, and DC = 4. What is the area of △ABC?



5. In the figure below, the given side lengths of △ABC are in inches. What is the are, in square inches, of △ABC?

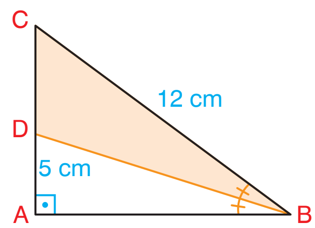


6. The rectangle shown in the figure below is partitioned into 3 triangles, 2 of which are shaded. What is the total area, in square inches, of the 2 shade regions?



7. According to the figure, [BD] is angle bisector of ∠ABC.

If |AD| = 5 cm and |BC| = 12 cm then find the are of △CDB.



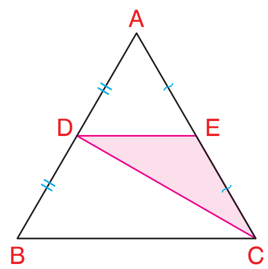
8. In the figure,

|AD| = |DB| and

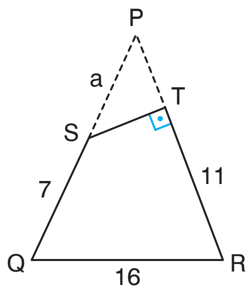
|AE| = |EC|.

If △(ABC) = 72 cm2

then find A(DCE).



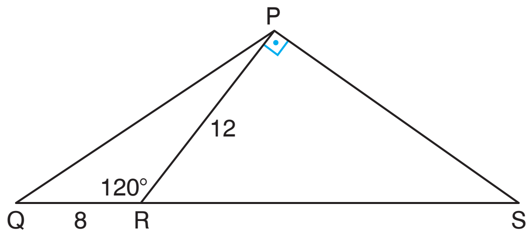
1. Isosceles triangle PQR has PQ = PR and QR = 16. Point S is on PQ and T is on PR so that ST is perpendicular to PR, ST = 8, TR = 11, and QS = 7. The area of quadrilateral STRQ is



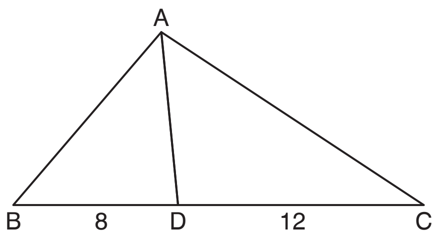
2. In the diagram, R is on QS and QR = 8.

Also, PR = 12, ∠PRQ = 120°, and ∠RPS = 90°.

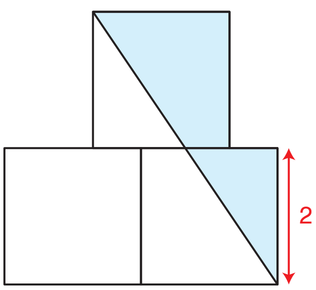
What is the are of △QPS?



3. The are of △ABC is 60 square units. If BD = 8 units and DC = 12 units, the area (in square units) of △ABD is



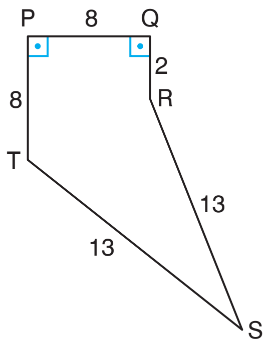
4.



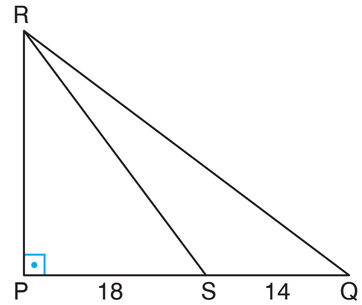
3 congruent sqaures

Shadow Area = ?

5. In the diagram, PQRST is a pentagon with PQ = 8, QR = 2, RS = 13, ST = 13, and TP = 8. Also, ∠TPQ = ∠PQR = 90°. What is the area of pentagon PQRST?



6. In △PQR, ∠RPQ = 90° and S is on PQ. IF SQ = 14, SP = 18, and SR = 30, then the are of △QRS is



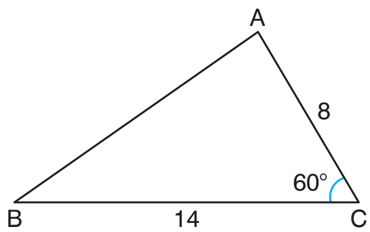
7. In a triangle ABC,

BC = 14 cm,

AC = 8 cm, and

m∠ACB = 60°.

What is the area of △ABC?



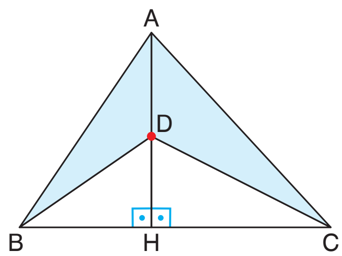
8. In the figure, point D lies on the altitude AH.

Given that

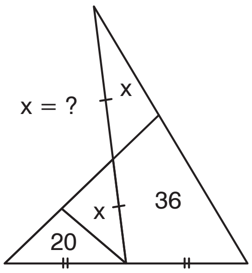
BC = 10, and

AD = 6,

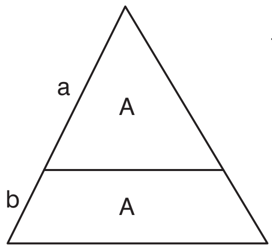
find the area of ABDC.



1.



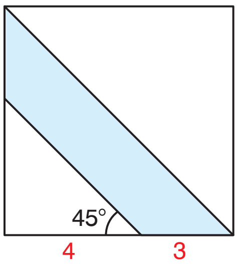
2. Equilateral triangle of two equal parts.



$\frac{a}{b} = ?$

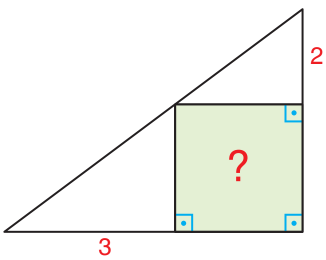
3.

Square

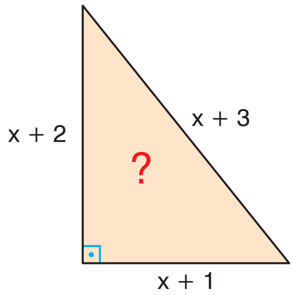


Area of shade ?

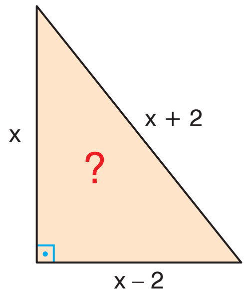
4.



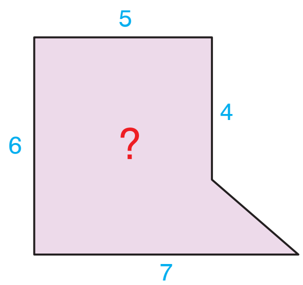
5.



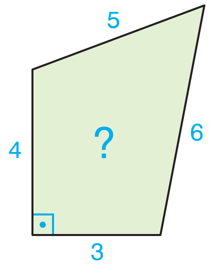
6.



7.

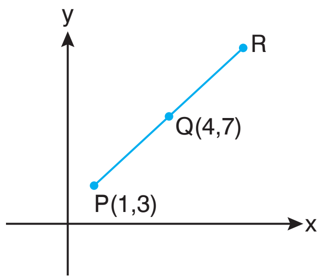


8.



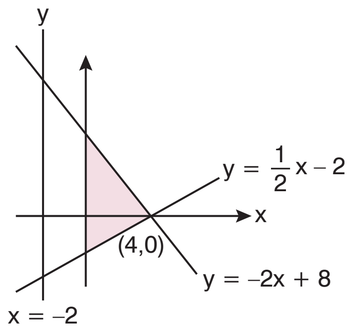
1. The vertices of a triangle are (1, 1), (5, 4) and (3, 4). Find the area of the triangle.

2. In the diagram, point Q is the midpoint of PR. The coordinates of R are



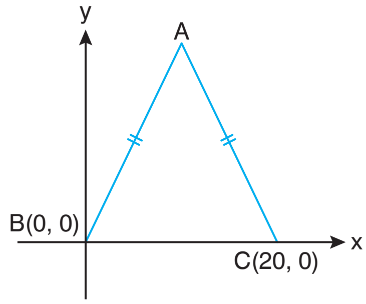
3. The lines $y = -2x + 8$ and $y = \frac{1}{2}x - 2$ meet at (4, 0), as shown.

What is the area of the triangle formed by these two lines and the line $x = -2$?

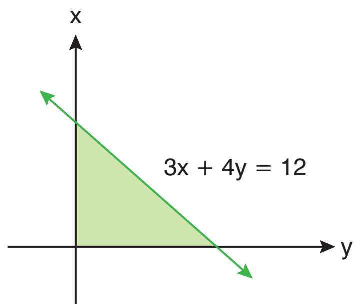


4. In the figure, ABC is an isosceles triangle, and its area is 180 square units.

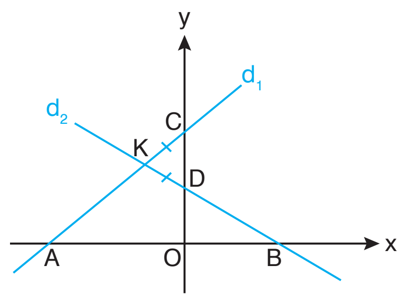
What is the y-coordinate of point A, if AB = AC?



5. What is the area bounded by the x-axis, y-axis, and $3x + 4y = 12$?

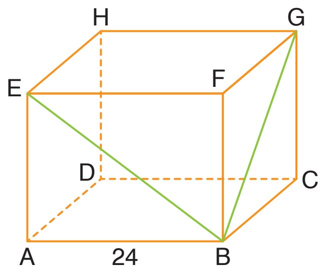


6. In the given figure, K is the intersection point of the lines d_1 and d_2.



A(–6, 0),
B(4, 0),
D(0, 2), and
CK = KD.
Area ΔKDC?

7.



The figure is a rectangular prism. If AB = 24 cm, EB = 25 cm, and BG = 25 cm, what is the volume of the prism?

8. A rectangular box has integer side lengths in the ratio of 2 : 3 :4. Which of the following could be the volume of the box?

A) 72 B) 96

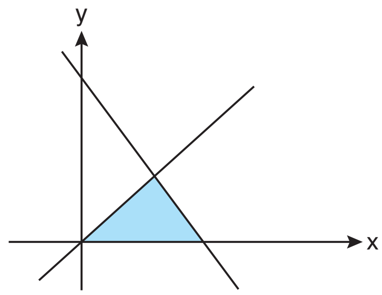
C) 120 D) 192

9. Which two-dimensional figure is always formed when a plane intersects a right cylinder perpendicular to its base?

(1) circle (2) rhombus

(3) triangle (4) rectangle

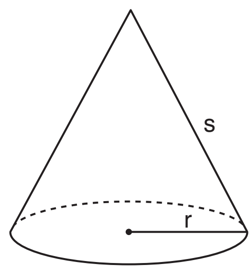
10. In the diagram, the shaded region is bounded by the x-axis and the lines $y = x$, and $y = -2x + 3$. The area of the shaded region is



11. Find the radius of the circle given by

$$x^2 + y^2 + 10x - 12y + 20 = 0$$

12.



The total surface area T of any right circular cone with a radius r and a slant height s can be determined using the formula

$T = \pi r^2 + \pi rs$.

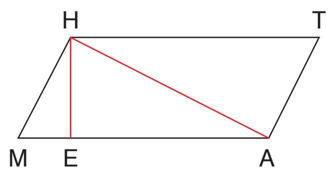
If a cone has a radius of 3 inches and slant height of 5 inches, what is its total surface area, in square inches?

Regents Geometry Exams: Altitude–hypotenuse similarity products (right triangles)

Problem 5

(NY Regents Geometry-Jun'19, #35)

Given: Quadrilateral MATH, $\overline{HM} \cong \overline{AT}$, $\overline{HT} \cong \overline{AM}$, $\overline{HE} \perp \overline{MEA}$, and $\overline{HA} \perp \overline{AT}$



Prove: TA · HA = HE · TH

Proof 1

Regular (paragraph) proof

Since HM ≅ AT and HT ≅ AM, both pairs of opposite sides of MATH are congruent, so MATH is a parallelogram. Hence

$$HT \parallel MA \text{ and } HM \parallel AT.$$

Because HE ⊥ MA and HT ∥ MA, it follows that HE ⊥ HT. Thus HE is the perpendicular distance between the parallel lines HT and MA. Point A lies on MA, so the altitude from A to the base HT in ΔAHT has length HE.

Triangle AHT is right at A (since HA ⊥ AT). Compute its area in two ways:

$$[\Delta AHT] = \frac{1}{2}(AT)(HA) \quad \text{(legs at the right angle)} = \frac{1}{2}(HT)(HE) \quad \text{(base HT, altitude HE).}$$

Equating the two expressions gives AT · HA = HT · HE, as required.

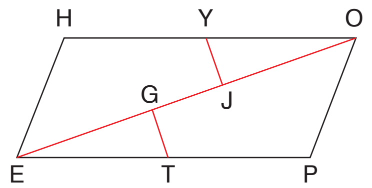
Proof 2

Statement	Reason
1. HM ≅ AT and HT ≅ AM	Given
2. MATH is a parallelogram	In a quadrilateral, both pairs of opposite sides congruent → parallelogram
3. HT ∥ MA and HM ∥ AT	Opposite sides of a parallelogram are parallel
4. HE ⊥ MA	Given (HE ⊥MEA)
5. HE ⊥ HT	If a line is perpendicular to one of two parallel lines, it's perpendicular to the other (3 & 4)
6. In ΔAHT, ∠HAT = 90°.	Given HA ⊥ AT
7. The altitude to base HT has length HE	HE ⊥ Ht and A ∈ MA ∥ HT; perpendicular distance between parallel lines is constant
8. $[\Delta AHT] = \frac{1}{2}(AT)(HA)$	Area of right triangle using legs (6)
9. $[\Delta AHT] = \frac{1}{2}(HT)(HE)$	Area with base HT and altitude HE(7)
10. AT · HA = HT · HE	Equate areas from (8) and (9)

Problem 6

(NY Regents Geometry-Jan'19, #35)

In quadrilateral HOPE below, $\overline{EH} \cong \overline{OP}$, $\overline{EP} \cong \overline{OH}$, $\overline{EJ} \cong \overline{OG}$, and $\overline{TG}$ and $\overline{YJ}$ are perpendicular to diagonal $\overline{EO}$ at points G and J, respectively.



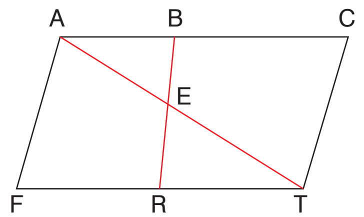
Prove that $\overline{TG} \cong \overline{YJ}$.

My Proof

Problem 7

(NY Regents Geometry-Aug'23, #35)

In the diagram below of quadrilateral FACT, $\overline{BR}$ intersects diagonal $\overline{AT}$ at E, $\overline{AF} \parallel \overline{CT}$, and $\overline{AF} \cong \overline{CT}$.



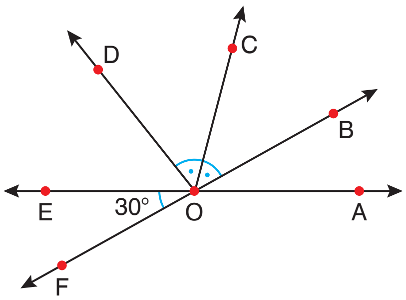
Prove: (AB)(TE) = (AE)(TR)

My Proof

GeoTopia 0.5 Post-Test & Evaluation

1. In the figure below, ∠FOE = 30° and ∠AOD = 140°. Ray OC is the bisector of ∠DOB.

Find the measure of ∠DOC.

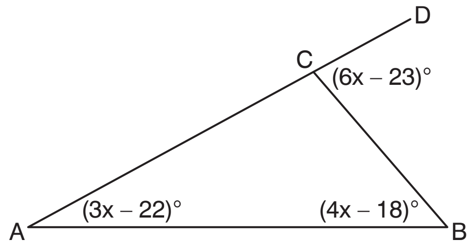


2. Two angles of an isosceles triangle measure 70 and x.

What is the sum of the three possible values of x?

3. **(NYSED, Geometry Regents Exam, June 2025, Q. 26)**

In △ABC below, $\overline{AC}$ is extended through C to D, m∠A = (3x – 22)° m∠B = (4x – 18)°, and m∠BCD = (6x – 23)°.



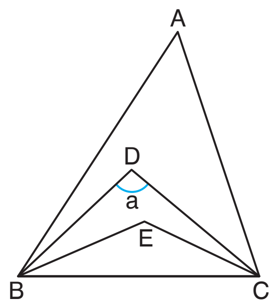
Determine and state m∠ACB.

4. In the given figure,

ABC is a triangle,

The trisection of angles B and C meet at points D and E, and m∠BAC = 30°.

What is m∠BDC = a?



1. 20°. ∠HFG = 130° so its acute supplement is 50°. Given ∠EHF = 70°, the angle that ray HE makes with the base is 70° − 50° = 20°. Since AB ∥ base, m∠ABE = 20°.

2. 36°. Let x = m∠BAC. Because AB = AC, we have m∠B = m∠C = (180° − x)/2. Using AD = BD = BC gives equal angles at those points; angle chasing leads to 5x = 180°, hence x = 36°.

3. 30°. AB = AC and m∠BAC = 40° so m∠B = m∠C = 70°. Let m∠ABD = x. In isosceles ΔDBC we get m°DBC = m°BDC = 70° − x. Using the triangle sum for ΔDBC yields x = 30°.

4. 45°. BC = CD = DA and m∠B = m∠C = 80°. Triangles BCD and CDA are isosceles; solving their angle sums forces the remaining angle at A to be 45°.

5. $4\sqrt{3}$. Right triangle with a 60° angle and hypotenuse 8 gives opposite side $4\sqrt{3}$. So AB = $4\sqrt{3}$.

6. 20°. On hypotenuse $\overline{SR}$ of right ΔSTR, SQ : QR = 1 : 4. Let SQ = x, QR = 4x. Altitude TQ satisfies $TQ^2 = SQ \cdot QR$: $8^2 = x \cdot 4x \Rightarrow x = 4$, so SR = 5x = 20.

7. 14°. NE is a midsegment, so NE = $\frac{1}{2}$GL. Given NE = 15 and GL = 3x − 12, we get $15 = \frac{1}{2}(3x - 12) \Rightarrow x = 14$.

8. $\frac{49}{2}$. In a triangle, centroid divides each median in ratio 2 : 1 from vertex. CE = 5 ⇒ CX = 10; Y F = 21 ⇒ CF = 7; XZ = 15 ⇒ XF = 7.5. So $P_{\Delta CFX}$ = CX + CF + XF = 10 + 7 + 7.5 = $\frac{49}{2}$.

9. 4. Big right triangle area is 12. The three small right triangles shown have total area 1+3+4 =8. Shaded area =12−8 = 4.

10. 4. ΔABC is right with legs 5, 12: area 30 and AC = 13. ΔACD has sides 13, 14, 15; with s = 21, its area (Heron) is 84. So [ABCD] = 30 + 84 = 114.

11. 114. Square interior angle = 90°, regular pentagon interior angle = 108°. Using the shared side and exterior/interior relations, the marked sum ∠CEB + ∠CBE is 18°.

12. 47.5°. With AB ∥ CD, diagonals intersecting, and given ∠MDA = 35°, ∠DCE = 25°, ∠NEC = 30°, angle chasing yields m∠ABD = 47.5°.

13. 13. Rectangle area = 5 · 6 = 30. Right ΔDCE has height 5; set $\frac{1}{2} \cdot CE \cdot 5 = 30 \Rightarrow CE = 12$.

Then DE = $\sqrt{12^2 + 5^2 = 13}$.

14. 5. In the square, AE = 8 and EC = 2. Using coordinates or Pythagoras on right triangles inside, one finds DE = 5.

15. 1800. Bases BC = 50, AE = 30, height 24. Area of trapezoid ABCD: $\frac{1}{2}$ (50 + 30) · 24 = 40 · 45 = 1800.

16. 108. Square ABCD with E midpoint of CD. Quadrilateral AFED has area 45. Let side be s. Then $45 = \frac{3}{4}s^2 \Rightarrow s^2 =$ 60, so total area ABCD = 4 · 27 = 108.